NBS REPORT

5575

# Measurements of the Power Spectrum of Fading in Tropospheric Scatter Propagation

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by

A. F. Barghausen, H. B. Janes, F. O. Guiraud,
S. Murahata and C. F. Peterson

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# NATIONAL BUREAU OF STANDARDS REPORT

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## I. Introduction

In many analyses of the time fluctuations in field strength observed over tropospheric scatter paths, the statistics obtained have consisted chiefly of median signal level, fading (interdecile) range, and fading rate, all measured over some arbitrarily-chosen time interval. Although the fading range gives a reasonable measure of the total variance of the signal envelope, interpretation of the fading rate is made difficult by the fact that it is a complicated function of 1) the net frequency response characteristic of the receiving and recording circuits, and 2) the distribution of the total variance over the range of frequencies being observed, i.e., the power spectrum.(In this report, possible confusion of carrier frequency and frequency of variation of the envelope will be avoided by referring to the former as "radio frequency" and to the latter simply as "frequency".)

In view of this difficulty in interpreting fading rates and ranges it seems desirable to compute the power spectrum of the envelope flucuations, i.e., the variance per unit bandwidth plotted as a function of frequency. The power spectrum should be a more useful tool in testing propagation theory against experiment because 1) it is relatively easy to compensate for the circuit response characteristics, making possible comparison of results obtained with different recording systems, and 2) by showing how much of the total variance was contributed by fluctuations in each of several frequency bands, it should be useful in determining scales of turbulence. With the development of high speed computers, the power spectrum is now not only a desirable statistic, but also a readily available one.

The purpose of this report is to present the results of a pilot study of the power spectrum of fading observed in tropospheric scatter signals, with particular emphasis on its relationship to fading rate and fading range.

## II. Description of Transmission Paths, Equipment and Measurements.

During the months of January, March, April, November and December, 1957, field intensity recordings were made at 1046 megacycles on two different tropospheric scatter paths in Colorado and Kansas. A map showing the location of the paths is presented in Fig. 1. Terrain profiles of the paths, plotted on the basis of a standard atmosphere with an earth's curvature assumed to be 4/3 of its actual value, are shown in Figs. 2 and 3. A summary of the pertinent parameters of these paths as obtained from the above figures are given in Table I below.

Table I

| Path | d | $h_{te}$ | $h_{re}$ | $\theta$ |
|---|---|---|---|---|
| Cheyenne Mt. - Garden City | 226.5 miles | 8805 ft. | 2865 ft. | 26.7 milliradians |
| Haswell-Boulder | | | | |
| (Table Mesa) | 165.8 miles | 4300 ft. | 5470 ft. | 33.1 milliradians |
| (Green Mt. Mesa) | 157.7 miles | 4300 ft. | 5820 ft. | 31.6 milliradians |

Here:

- $d$ = distance between transmitter and receiver in statute miles.
- $h_{te}$ = transmitting antenna height above mean sea level, in feet.
- $h_{re}$ = receiving antenna height above mean sea level, in feet.
- $\theta$ = angular distance or the angle, in milliradians, between tangent lines to the horizons of the receiving and transmitting antennas as indicated in Figs. 2 and 3.

Some of the pertinent characteristics of the equipment used for the tests are outlined in Table II below.

Table II

| Path | Cheyenne Mt. - Garden City | Haswell - Boulder |
|---|---|---|
| Receiver Bandwidth | 760 cycles | 760 cycles |
| Receiver Noise Figure | 15 db | 15 db |
| Receiving Antenna Gain* | 26 db | 29.9 db |
| Transmitting Antenna Gain* | 26 db | 29.9 db |
| Line Losses | 3.0 db | 3.2 db |
| Transmitter Power | 4000 watts | 200 watts |

* over isotropic radiator

The amplitude of the received fields were recorded on both Esterline-Angus and Sanborn direct-writing recorders. The filter characteristics of the recording systems are shown in Fig. 4.

Measurements were made of the amplitude variations of the received field strength during selected intervals within the morning, afternoon and evening hours of various days for the months given above.

From these recordings, samples were selected to include examples of the extremes in observed fading rates as well as the various times of day at which samples were recorded.

## III. Analysis of Data

Twenty-one 100-second periods were selected from all of the data. These periods are tabulated below in Table III, which indicates the observed fading rate, fading range and median level. The fading rate is defined as the number of times per second that the envelope of the received field crosses the median level with a positive slope 1/. The fading range is defined as the ratio, in decibels, of the signal level exceeded 10 per cent of the time interval being analyzed to that exceeded 90 per cent of the time interval 2/.

Table III

| Date | Sample No. | Location | Initial Time | Median* | Fading Rate | Fading Range |
|---|---|---|---|---|---|---|
| 1/30/57 | 4 | Garden City | 8:00 p.m. | 230.8 db | 2.60 cycles/sec. | 15.7 db |
| 1/31/57 | 5 | Garden City | 6:40 a.m. | 229.5 db | 3.05 cycles/sec. | 13.2 db |
| 1/31/57 | 6 | Garden City | 12:25 p.m. | 230.3 db | 2.41 cycles/sec. | 9.8 db |
| 3/6 /57 | 12 | Garden City | 8:50 p.m. | 223.8 db | 2.15 cycles/sec. | 14.0 db |
| 3/7 /57 | 13 | Garden City | 6:00 a.m. | 227.7 db | 4.10 cycles/sec. | 20.1 db |
| 3/7 /57 | 14 | Garden City | 12:15 p.m. | 219.4 db | 2.70 cycles/sec. | 16.0 db |
| 3/7 /57 | 15 | Garden City | 3:10 p.m. | 210.9 db | 3.80 cycles/sec. | 17.9 db |
| 4/16/57 | 17 | Garden City | 5:15 p.m. | 218.5 db | 3.34 cycles/sec. | 12.5 db |
| 4/17/57 | 21 | Garden City | 7:30 a.m. | 213.9 db | 1.50 cycles/sec. | 13.7 db |
| 4/17/57 | 22 | Garden City | 10:10 a.m. | 216.2 db | 0.89 cycles/sec. | 15.9 db |
| 4/17/57 | 23 | Garden City | 4:55 p.m. | 207.8 db | 0.67 cycles/sec. | 14.0 db |
| 4/18/57 | 25 | Garden City | 6:10 a.m. | 198.0 db | 1.23 cycles/sec. | 15.3 db |
| 4/18/57 | 26 | Garden City | 12:49 p.m. | 218.6 db | 3.95 cycles/sec. | 17.7 db |
| 4/18/57 | 27 | Garden City | 6:30 p.m. | 224.5 db | 5.30 cycles/sec. | 14.5 db |
| 11/20/57 | 29 | Boulder | 9:20 p.m. | 225.1 db | 1.50 cycles/sec. | 18.4 db |
| 11/21/57 | 33 | Boulder | 6:30 a.m. | 219.8 db | 0.44 cycles/sec. | 9.9 db |
| 11/21/57 | 34 | Boulder | 2:08 p.m. | 221.6 db | 0.73 cycles/sec. | 15.0 db |
| 11/21/57 | 35 | Boulder | 8:50 p.m. | 229.4 db | 0.55 cycles/sec. | 17.2 db |
| 11/22/57 | 38 | Boulder | 3:00 a.m. | 222.8 db | 0.69 cycles/sec. | 12.0 db |
| 11/22/57 | 39 | Boulder | 10:10 a.m. | 234.1 db | 0.76 cycles/sec. | 14.8 db |
| 12/5 /57 | 43 | Boulder | 1:20 p.m. | 223.1 db | 3.13 cycles/sec. | 12.1 db |

* basic transmission loss

From these samples, seven periods were selected for a power spectrum analysis. Here again, these samples were chosen to include periods of both high and low fading rates. Fig. 5 and Fig. 6 show the cumulative distributions obtained for the seven selected periods, and from these graphs it is evident that, with the exception of run 13, they approximate the Rayleigh distribution. Fig. 7 shows two samples of the recordings obtained on each path. These do not coincide with the samples used in the analysis, but were selected to show the appearance of the record during periods of high and low fading rate. It is not known whether the rapid, small-amplitude fluctuations (especially evident in the Haswell-Boulder data) are contributed by the propagation mechanism or by circuit noise. In any event, they are too small and blurred to be accurately scaled and hence were ignored in determining the fading rate.

The input to the power spectrum computation for each sample consists of approximately 600 equally-spaced values of signal amplitude expressed in terms of microvolts across the receiver input terminals. The computation consists of obtaining the autocovariance function for lags from 0 through 30, computing its Fourier cosine transform, and modifying the latter in accordance with a method proposed by Blackman and Tukey [3]. The output of the process consists of 30 estimates of spectral density equally-spaced on the frequency scale from $\frac{1}{60\delta}$ cycles per second to $\frac{1}{2\delta}$ cycles per second, where $\delta$ is the data sampling interval in seconds. The spectral density has the dimensions variance per unit bandwidth, or in this case, $(\text{microvolts})^2$ per cycle per second.

Table IV gives the length of each sample, sampling interval, median signal level, fading rate, and total variance. In general, the sampling interval, $\delta$, was chosen so that in each case the spectrum would extend to include the highest frequencies observable in the record. (As noted below, spectra labeled 27(A) and 34(A) are exceptions to this rule.) The length of each sample is in general less than the 100 seconds used in obtaining the cumulative distributions, and was chosen to take into account the lowest frequencies of variation observable by visual inspection of the record.

Table IV

Information Relative to Samples Used in Spectrum Analysis

| Sample No. | Length (sec.) | Sampling Interval (sec.) | Median Signal Level* | Fading Rate** | Total Variance $(\mu v)^2$ |
|---|---|---|---|---|---|
| 13 | 12.6 | 0.02 | $5.4 \times 10^{-12}$ | 4.10 | 2.740 |
| 21 | 61.1 | 0.1 | $1.3 \times 10^{-10}$ | 1.50 | 1.258 |
| 22 | 62.1 | 0.1 | $7.9 \times 10^{-11}$ | 0.89 | 1.271 |
| 25 | 60.3 | 0.1 | $5.0 \times 10^{-9}$ | 1.23 | 73.02 |
| 27 | 12.5 | 0.02 | $1.1 \times 10^{-11}$ | 5.30 | 0.1255 |
| 27a | 60.6 | 0.1 | | 5.30 | 0.1377 |
| 29 | 61.4 | 0.1 | $2.2 \times 10^{-12}$ | 1.50 | 0.0204 |
| 34 | 61.1 | 0.1 | $6.3 \times 10^{-12}$ | 0.73 | 0.1020 |
| 34a | 312.0 | 0.5 | | 0.73 | 0.0955 |

* in watts at receiver terminals. The values and the fading rates were determined for the 100-second periods, but differ negligibly from the values for these samples.

** in fades per second.

Figure 8 shows the power spectra of the seven data samples. To obtain spectra 27(A) and 34(A), samples 27 and 34 were rescaled, this time increasing the length to extend the spectrum to lower frequencies. To conserve computing time, the sampling interval, $\delta$, was also increased. This results in some "aliasing", or adding into the computed spectrum the power actually present at the higher frequencies which were purposely ignored. The effects of aliasing are most noticeable at the high frequency end of 27(A) and 34(A), where they should disagree with 27 and 34 by about 2 to 1. This extension to lower frequencies shows that the sample spectra tend to level off and that the total variance increases very little with increasing sample length. From this, it appears that any large low frequency power densities in the true spectrum must occur at frequencies far below the portion of the spectrum shown here.

It will be noted that although the spectra are very similar in shape, they are spread over a wide range of power densities and also separated frequency-wise. If we assume that these samples all represent scatter-propagated signals, then they should all be Rayleigh distributed, and in fact from Figs. 5 and 6, this seems to be true for all of them except 13.

For a Rayleigh distributed signal, the variance of the envelope, i.e. the area under the spectrum, will be proportional to the median radio frequency signal power. So to compare the shape of these spectra, we can normalize them by dividing their ordinates by the median signal power of the corresponding 100-second sample in milliwatts. Similarly, if the shape of the spectra is preserved as the fading rate changes, their position on the frequency scale should be a function of fading rate.

Figure 9 shows the spectra replotted with the ordinates normalized as described above, and with the abscissas normalized by dividing by the fading rate. The similarity in shape of these spectra and the fact that (except for the non-Rayleigh sample 13) they are closely grouped even though they were recorded at widely-differing times and over two different paths suggests that at least for a given path, radio frequency and antenna system (1) the slope of the spectrum of envelope variations in scatter signals may be nearly constant in this frequency range, and (2) it may be possible to anticipate the major differences between the spectra of two Rayleigh-distributed data samples simply by determining their respective fading rates and median signal levels. These hypotheses will be tested by further experiments which will also supply information on the extent to which the field strength power spectrum is a function of path characteristics, radio frequency and antenna aperture area.

## References


1/ K. A. Norton, P. L. Rice, H. B. Janes, A. P. Barsis, "The Rate of Fading in Propagation Through a Turbulent Atmosphere", Proc. IRE, Vol. 43, pp. 1341-1353, October 1955.

2/ K. A. Norton, L. E. Vogler, W. V. Mansfield, P. J. Short, "The Probability Distribution of the Amplitude of a Constant Vector Plus a Rayleigh-Distributed Vector", Proc. IRE, Vol. 43, pp. 1354-1361, October 1955.

3/ R. B. Blackman, J. W. Tukey, "The Measurement of Power Spectra From the Point of View of Communications Engineering, Part I and Part II", The Bell System Technical Journal, Vol. XXXVII, No. 1, pp. 185-282, January 1958, and No. 2, pp. 485-569, March 1958.

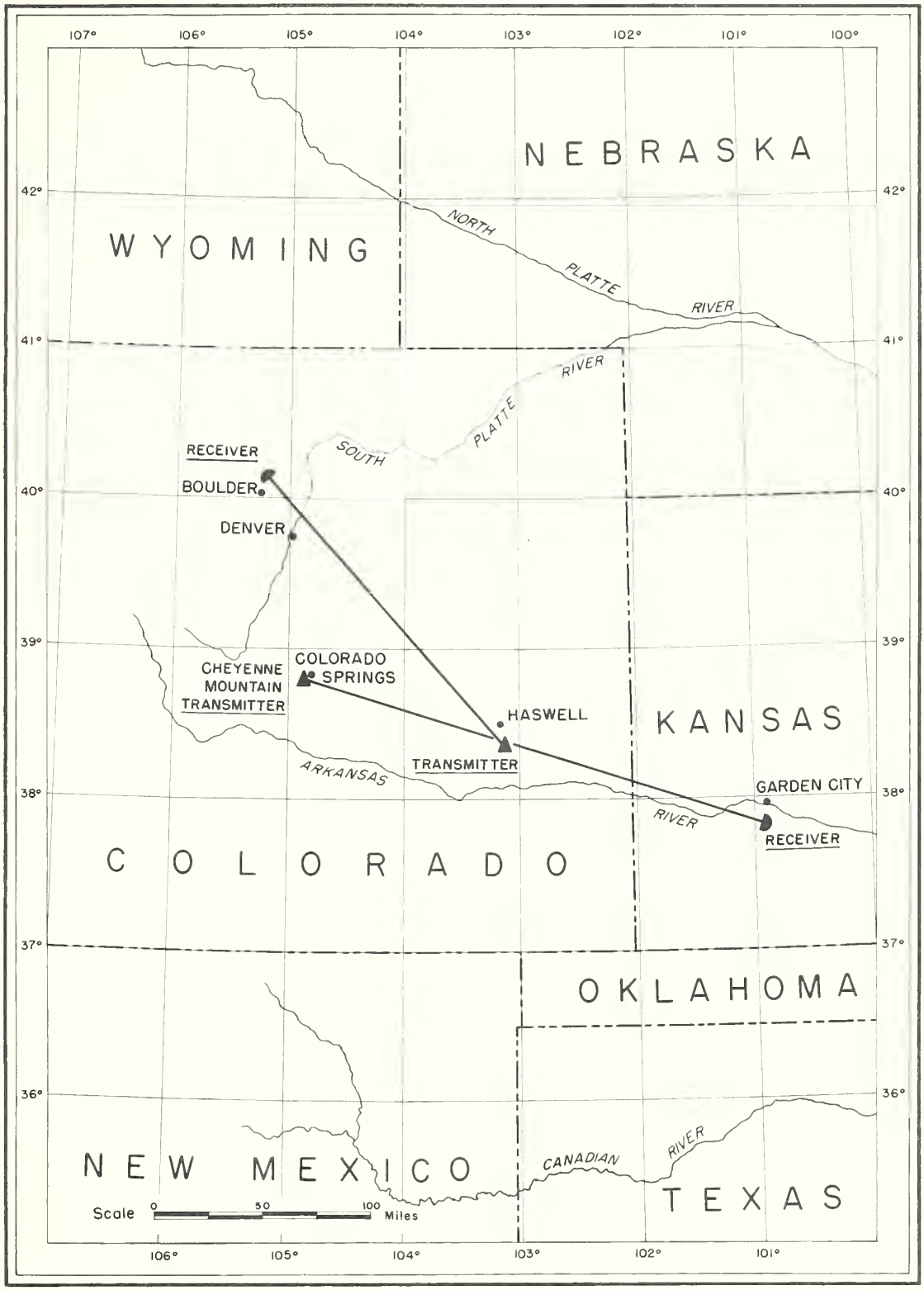

107°
106°
105°
104°
103°
102°
101°
100°
42°
41°
40°
39°
38°
37°
36°
NEBRASKA
WYOMING
NORTH PLATTE RIVER
SOUTH PLATTE RIVER
RECEIVER
BOULDER
DENVER
CHEYENNE MOUNTAIN TRANSMITTER
COLORADO SPRINGS
HASWELL
TRANSMITTER
KANSAS
ARKANSAS RIVER
GARDEN CITY
RECEIVER
COLORADO
OKLAHOMA
NEW MEXICO
CANADIAN RIVER
TEXAS
Scale 0 50 100 Miles

Figure 1

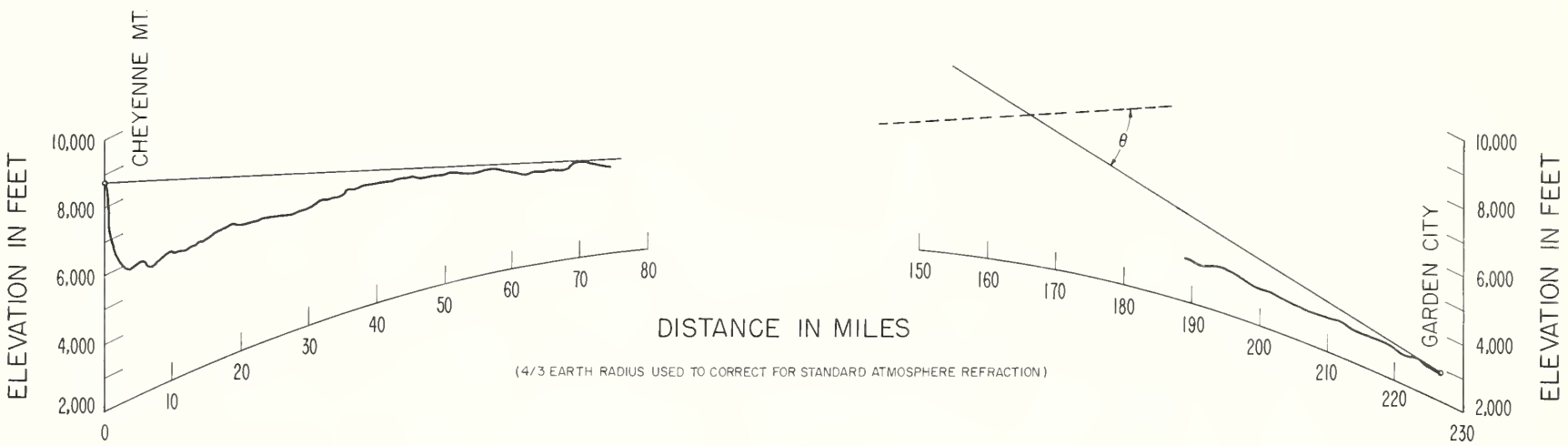

CHEYENNE MT. – GARDEN CITY TRANSMISSION PATH
ELEVATION IN FEET
CHEYENNE MT.
10,000
8,000
6,000
4,000
2,000
0
10
20
30
40
50
60
70
80
θ
150
160
170
180
190
200
210
220
230
GARDEN CITY
DISTANCE IN MILES
(4/3 EARTH RADIUS USED TO CORRECT FOR STANDARD ATMOSPHERE REFRACTION)

Figure 2

HASWELL - BOULDER TRANSMISSION PATHS

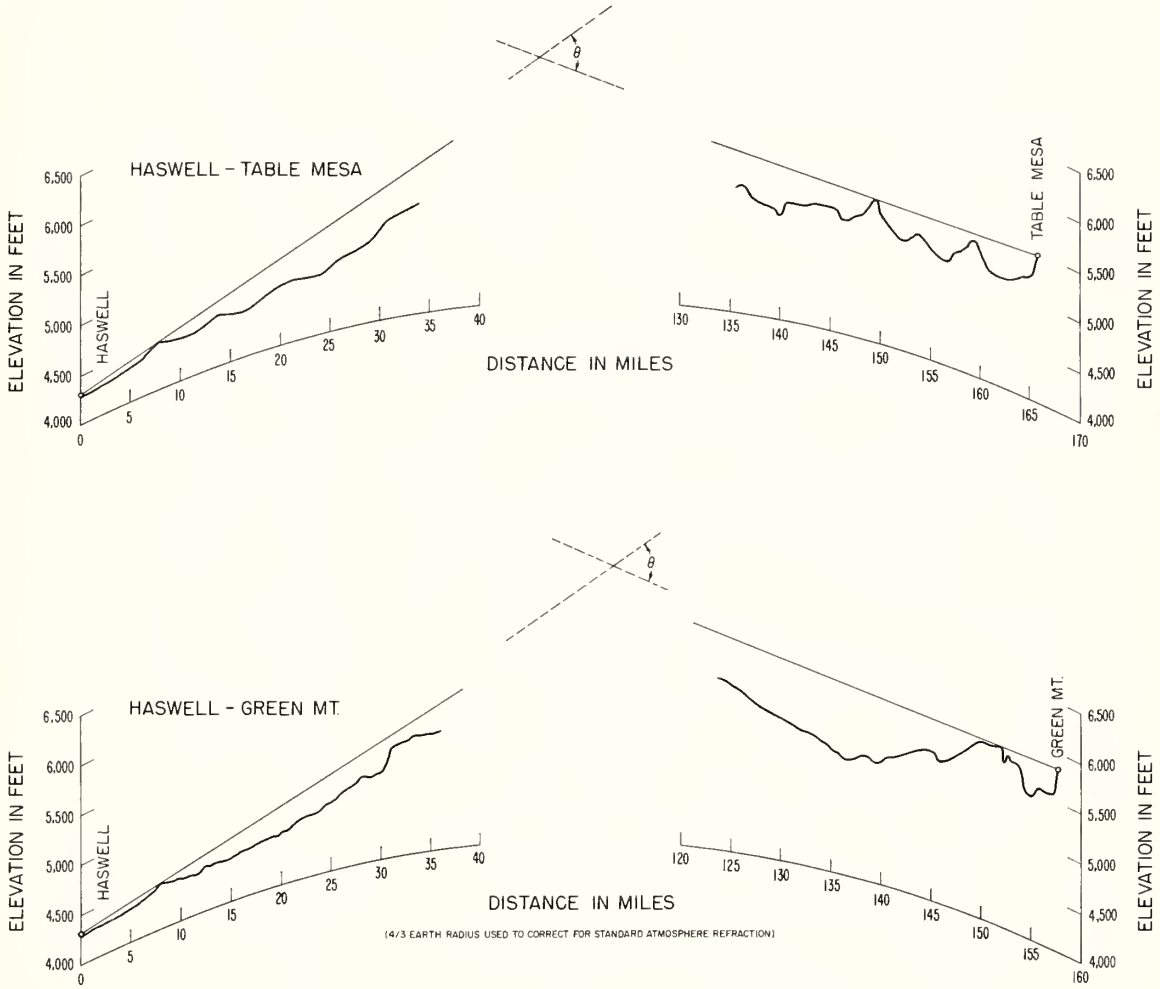



Figure 3

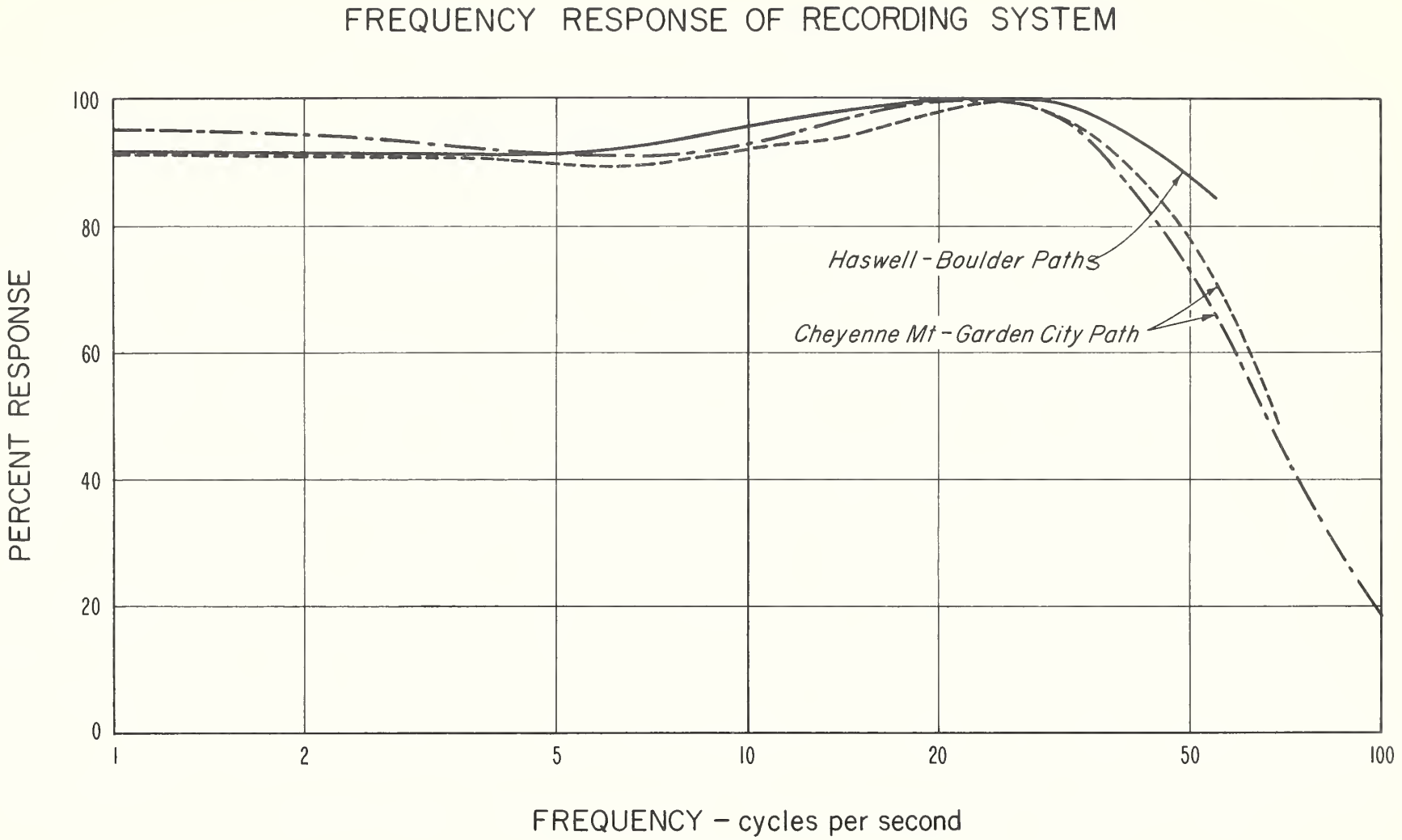

FREQUENCY RESPONSE OF RECORDING SYSTEM
PERCENT RESPONSE
100
80
60
40
20
0
1
2
5
10
20
50
100
Haswell - Boulder Paths
Cheyenne Mt - Garden City Path
FREQUENCY – cycles per second

Figure 4

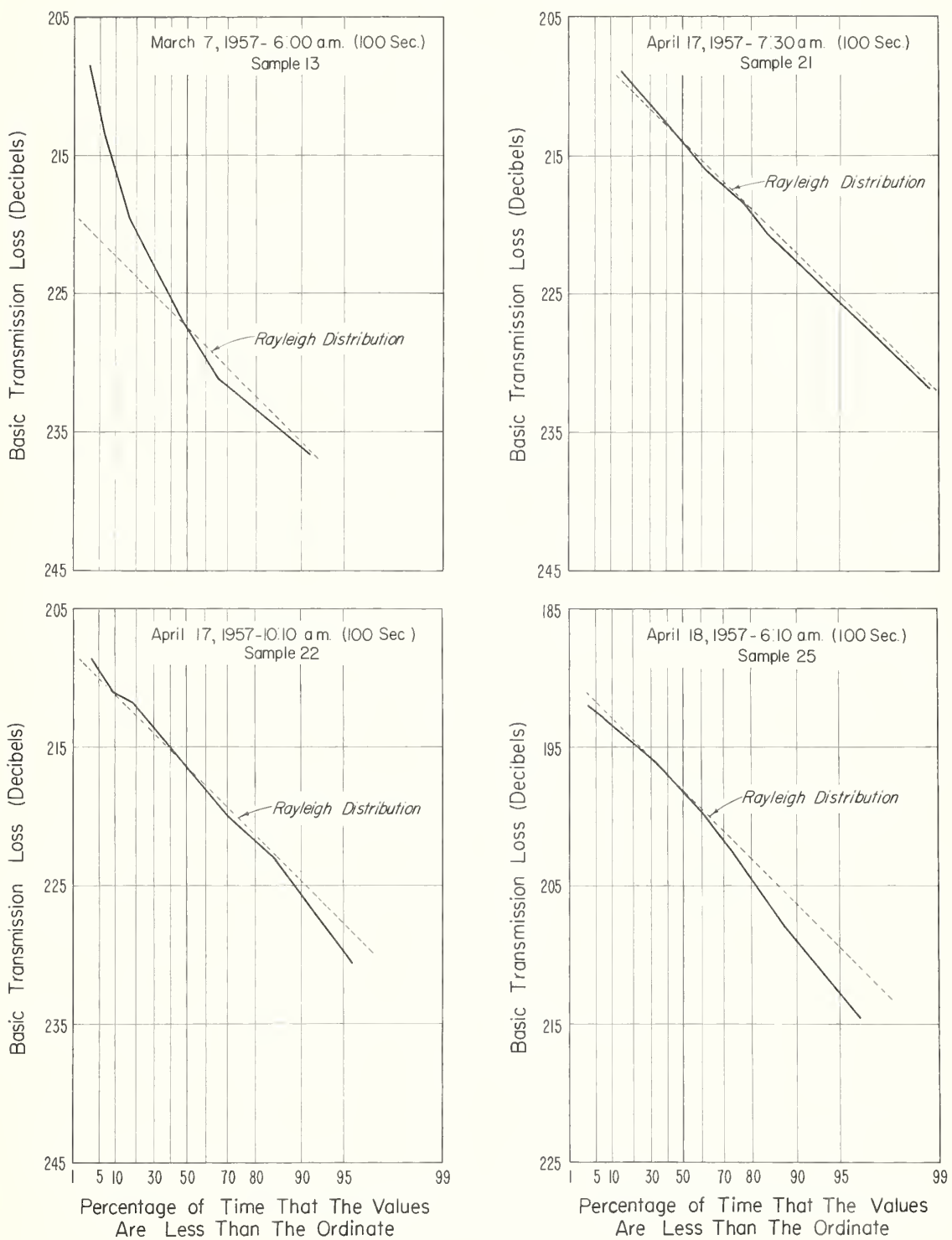

CUMULATIVE DISTRIBUTION OF BASIC TRANSMISSION LOSS
1046 Mc CHEYENNE MT. - GARDEN CITY PATH
March 7, 1957 - 6:00 a.m. (100 Sec.)
Sample 13
April 17, 1957 - 7:30 a.m. (100 Sec.)
Sample 21
April 17, 1957 - 10:10 a.m. (100 Sec.)
Sample 22
April 18, 1957 - 6:10 a.m. (100 Sec.)
Sample 25
Rayleigh Distribution
Basic Transmission Loss (Decibels)
205
215
225
235
245
185
195
1 5 10 30 50 70 80 90 95 99
Percentage of Time That The Values
Are Less Than The Ordinate

Figure 5

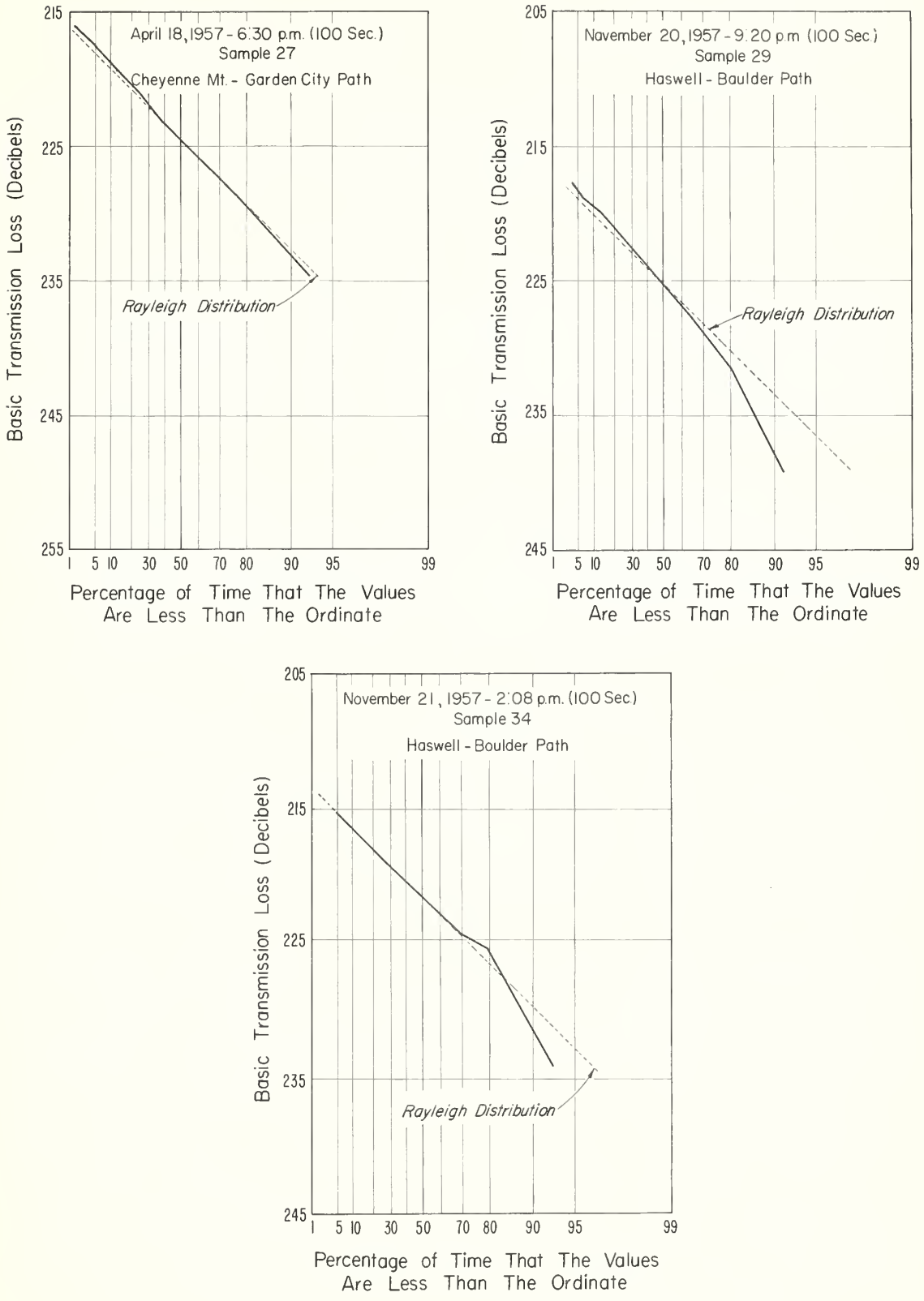

CUMULATIVE DISTRIBUTION OF BASIC TRANSMISSION LOSS 1046 Mc
April 18, 1957 - 6:30 p.m. (100 Sec.)
Sample 27
Cheyenne Mt. - Garden City Path
Rayleigh Distribution
Basic Transmission Loss (Decibels)
215
225
235
245
255
1 5 10 30 50 70 80 90 95 99
Percentage of Time That The Values Are Less Than The Ordinate
November 20, 1957 - 9.20 p.m. (100 Sec.)
Sample 29
Haswell - Baulder Path
Rayleigh Distribution
Basic Transmission Loss (Decibels)
205
215
225
235
245
1 5 10 30 50 70 80 90 95 99
Percentage of Time That The Values Are Less Than The Ordinate
November 21, 1957 - 2:08 p.m. (100 Sec.)
Sample 34
Haswell - Boulder Path
Rayleigh Distribution
Basic Transmission Loss (Decibels)
205
215
225
235
245
1 5 10 30 50 70 80 90 95 99
Percentage of Time That The Values Are Less Than The Ordinate

Figure 6

# SIGNAL ENVELOPE OF RECEIVED FIELDS
# 1046 MEGACYCLES

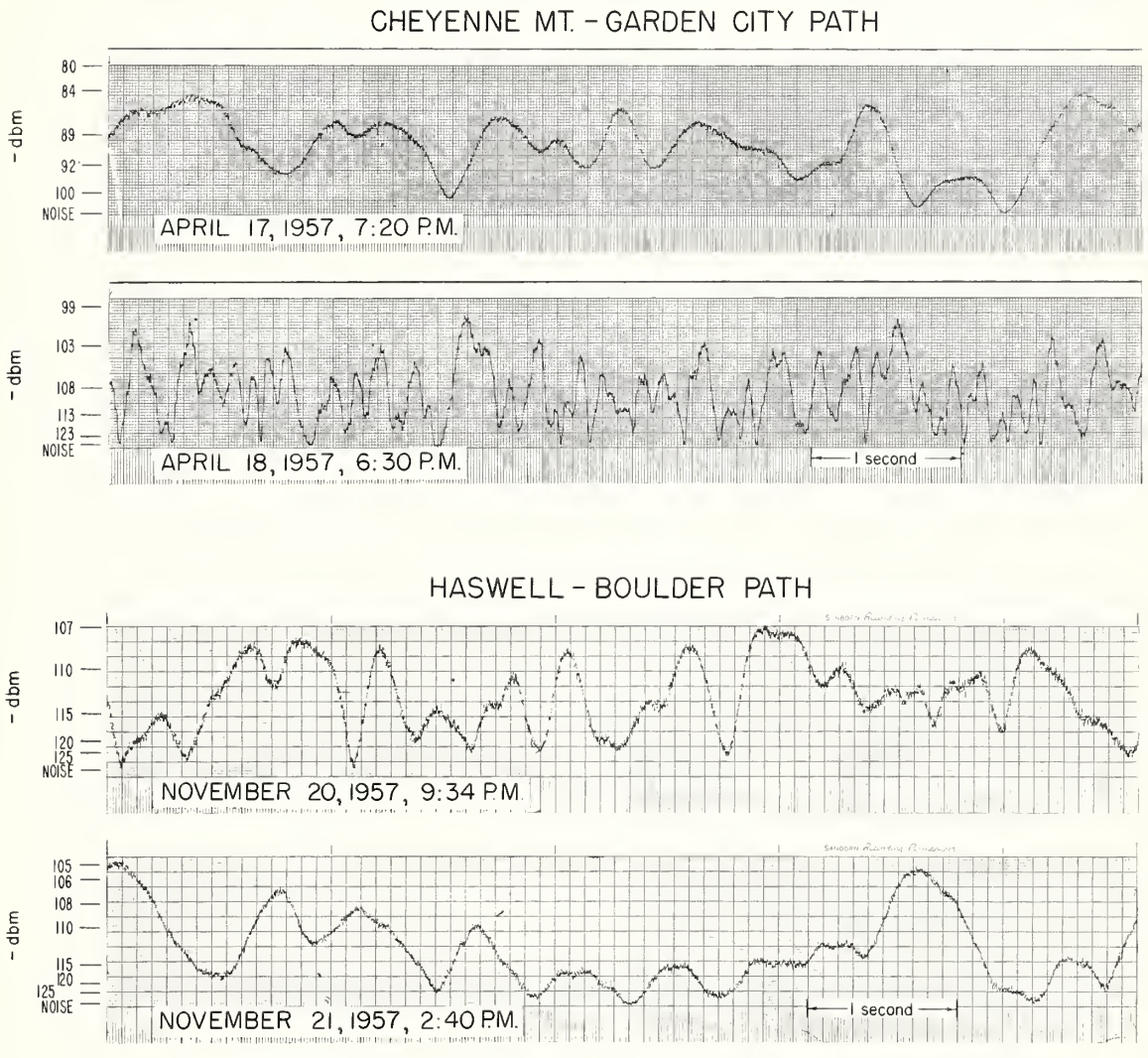



Figure 7

# POWER SPECTRUM OF VARIATIONS IN FIELD STRENGTH AT 1046 MEGACYLES

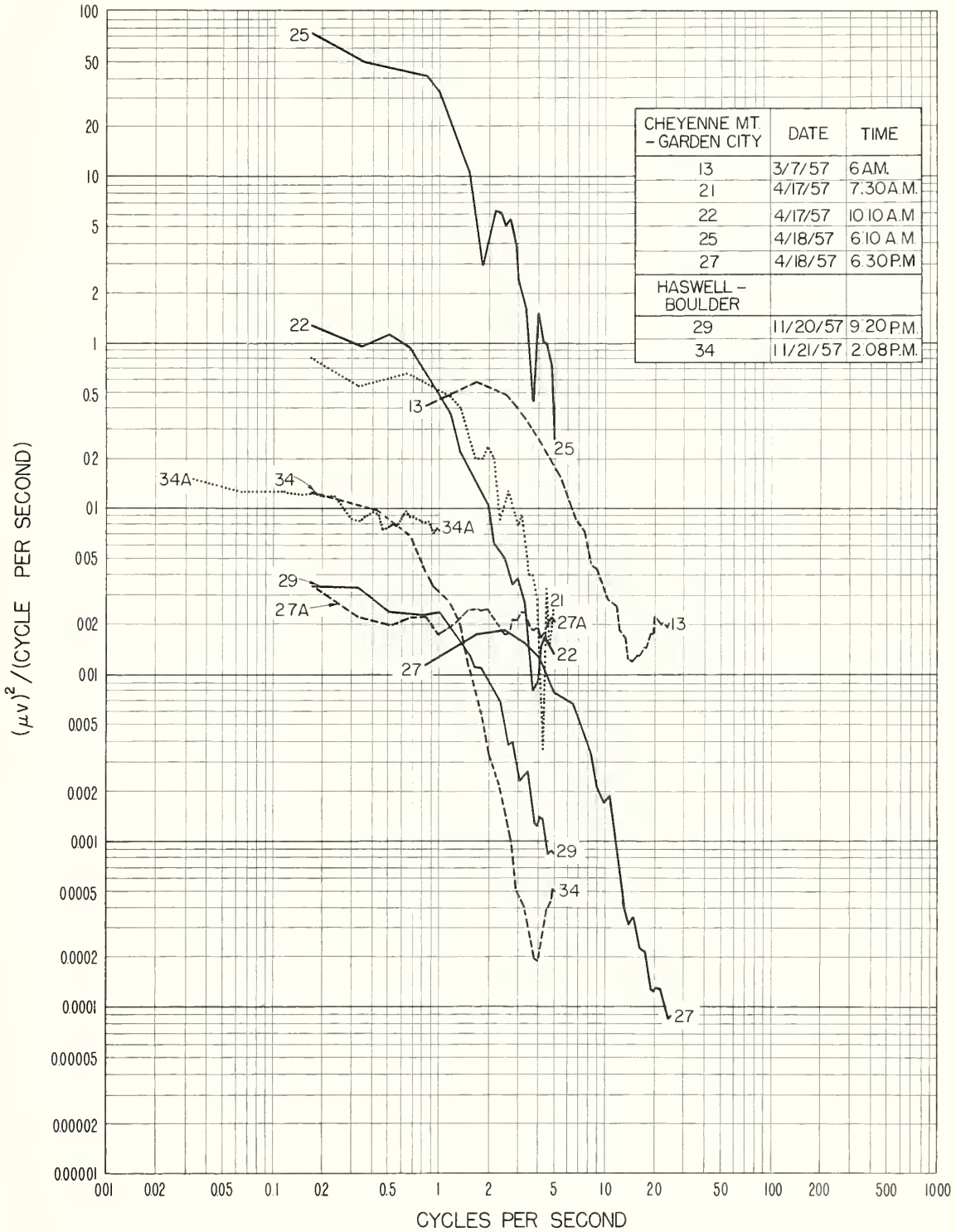
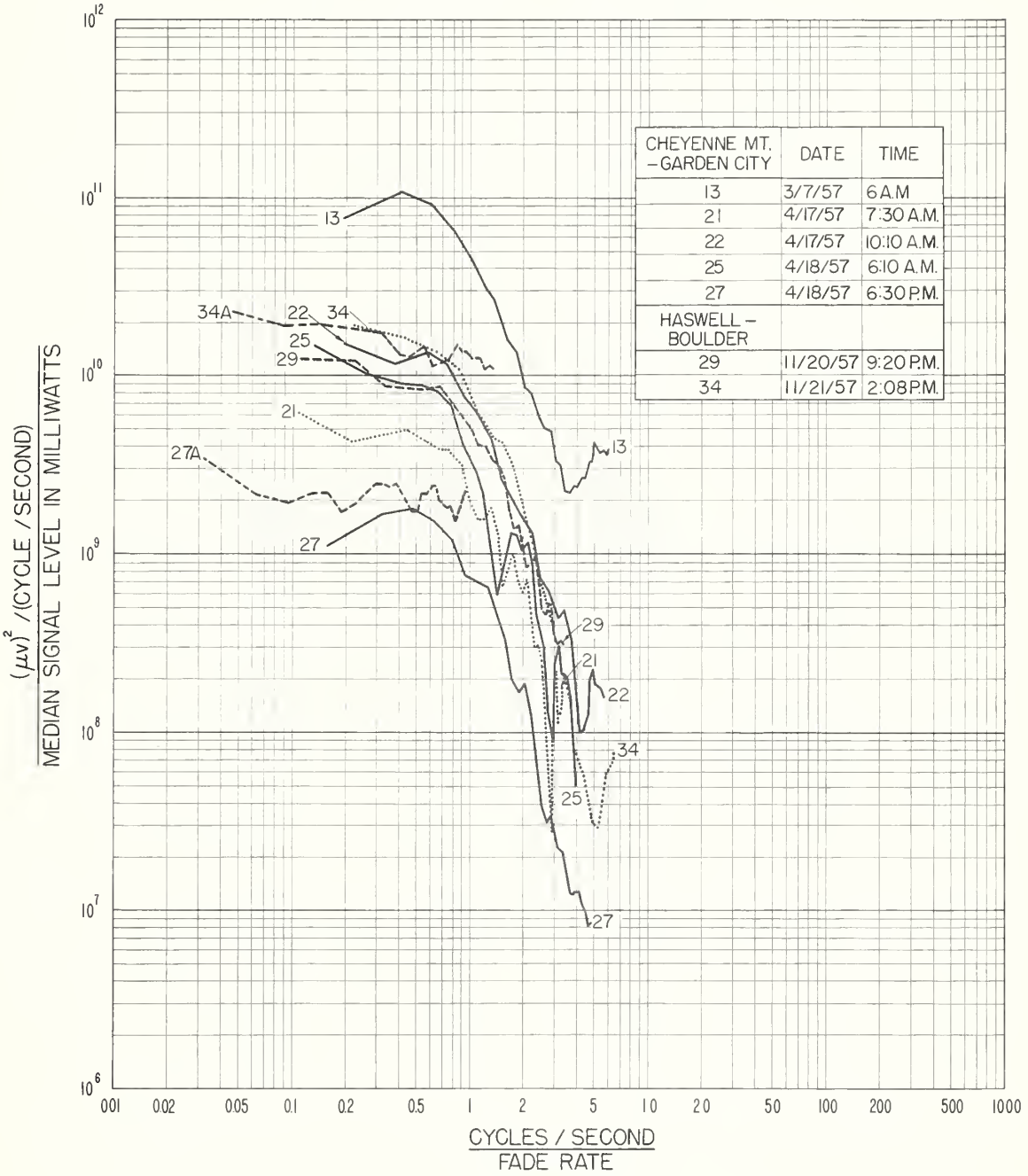



Figure 8

# NORMALIZED POWER SPECTRUM OF VARIATIONS IN FIELD STRENGTH AT 1046 MEGACYCLES




| CHEYENNE MT. – GARDEN CITY | DATE | TIME |
|---|---|---|
| 13 | 3/7/57 | 6 A.M |
| 21 | 4/17/57 | 7:30 A.M. |
| 22 | 4/17/57 | 10:10 A.M. |
| 25 | 4/18/57 | 6:10 A.M. |
| 27 | 4/18/57 | 6:30 P.M. |
| HASWELL – BOULDER | | |
| 29 | 11/20/57 | 9:20 P.M. |
| 34 | 11/21/57 | 2:08 P.M. |

Figure 9

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